Ninth Annual Upper Peninsula High School Math Challenge

Northern Michigan University (Marquette MI, USA)

Saturday April 14, 2018

RELAY 1 Category: PRIME NUMBERS PLAYER 1

What is the smallest two-digit prime number that does not remain prime when the digits are reversed?

Pass your answer to PLAYER 2.

$11 \Rightarrow 11$, nope.

$13 \Rightarrow 31$, nope,

$17 \Rightarrow 71$, nope.

$19 \Rightarrow 91 = 7 \times 13$ AHA

$\boxed{19}$

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RELAY 1 Category: POLYGONS PLAYER 2

You will receive a two-digit number from PLAYER 1. Multiply the digits together and take the (positive) square root of this product. Call this number n.

A p-sided polygon contains n right angles. Each of the other angles measures 135°. What is p?

Pass your answer to PLAYER 3.

Player 1: 19

$n = \sqrt{1 \times 9} = \sqrt{9} = 3$

Total angle : $(p-2)180$

n of these are 90

$p-n$ of these are 135.

$(p-2)180 = 90n + 135(p-n)$

$180p - 360 = 90n + 135p - 135n$

$45p = 360 - 45n$

$p = 8 - n$

$n = 3$ so $p = \boxed{5}$

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RELAY 1 Category: SUM AND PRODUCT OF THE ROOTS PLAYER 3

Let r be the number that you receive from PLAYER 2.

Let a and b be the two distinct roots of $x^2 - rx + r = 0$. What is the value of $a^2 + b^2$?

Pass your answer to PLAYER 4.

Player 2: 5

In $ax^2+bx+c=0$

$x_1+x_2 = -\frac{b}{a}$ $\qquad x_1 x_2 = \frac{c}{a}$

$(x_1+x_2)^2 = \frac{b^2}{a^2}$

$x_1^2+2x_1x_2+x_2^2 = \frac{b^2}{a^2}$

$x_1^2+x_2^2 = \frac{b^2}{a^2} - 2x_1x_2 = \frac{b^2}{a^2} - \frac{2c}{a}$

$a=1$ $b=-r$ $c=r$

$x_1^2+x_2^2 = r^2-2r = r(r-2)$

$r=5$, so $x_1^2+x_2^2 = 5\cdot 3 = \boxed{15}$

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RELAY 1 Category: PROBABILITY PLAYER 4

The number you will receive from PLAYER 3 is the number of balls in a bin.

2/3 of the balls in the bin are red, and 1/3 of them are blue. If I grab two balls at the same time from the bin, what is probability that both of them are blue?

Run your answer to the front.

Player 3: 15

So, 10 red balls, 5 blue

Probability is $\frac{5}{15} \cdot \frac{4}{14} = \frac{1}{3} \cdot \frac{2}{7} = \boxed{\frac{2}{21}}$

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RELAY 2 Category: CIRCLES PLAYER 1

I have a wheel two feet in diameter and I start it rolling. After the wheel has traveled a mile (5280 ft), how many **complete** revolutions has the wheel made?

(Hint #1: Your answer should be an integer.)

(Hint #2: Consider using the approximation $\pi \approx \frac{22}{7}$.)

Pass your answer to PLAYER 2.

diameter is 2, so radius is 1.

$C = 2\pi r \approx 2 \cdot \frac{22}{7} \cdot 1 = \frac{44}{7}$

So, to go 5280 ft. it makes $\frac{5280}{44/7}$, $\frac{5280 \cdot 7}{44} = 120 \cdot 7 = \boxed{840}$

The actual answer is $\frac{5280}{2\pi} = 840.3380995\ldots$

so the answer really is 840.

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RELAY 2 Category: LINES AND SLOPES PLAYER 2

Let the number you receive from PLAYER 1 be k.

A line of slope 42 passes through the point $(5, k)$. Another point on the line has coordinates $(8, y)$. What is the value of y?

Pass your answer to PLAYER 3.

Player 1: 840

$k = 840$

pt: $(5, 840)$ $m = 42$

$y - 840 = 42(x-5)$

$y - 840 = 42x - 210$

$y = 42x + 630$

when $x = 8$, $y = 42(8) + 630 = \boxed{966}$

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RELAY 2 Category: ARABIC NUMERALS PLAYER 3

Let n be the number you receive from PLAYER 2.

Let m be the smallest integer greater than n with the property that m reads the same when m is rotated upside down. What is the difference $m - n$?

Pass your answer to PLAYER 4.

Player 2: 966

m must be 986.

$986 - 966 = \boxed{20}$

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RELAY 2 Category: AGE PROBLEM PLAYER 4

The number you will receive from PLAYER 3 is half of my age now.

In ten years, my sister will be exactly half of my age at that time. How old is my sister now?

Run your answer to the front.

Player 3 : 20

Let k be this number;

m : my age $= 2k$

S : sister's age

$$S+10 = \frac{1}{2}(m+10)$$

$$S+10 = \frac{1}{2}m+5$$

$$S = \frac{1}{2}\cdot m - 5$$

$$S = k-5$$

$k=20$, so $S = \boxed{15}$

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RELAY 3 Category: INFINITE SERIES PLAYER 1

I drop a rubber ball from a height of ten feet. Each time it bounces it reaches 90% of its previous height. When the ball finally comes to rest, what is the distance, in feet, it has traveled?

Pass your answer to PLAYER 2.

The first time it bounces, it moves down 10 ft and up 9 so it moves 19 ft.

Each subsequent bounce has it move 90% of the previous bounce.

The distance is $19 + (0.9)19 + (0.9)^2 19 + (0.9)^3 19 + \ldots$

The formula is

$$S = \frac{9}{1-r} = \frac{19}{1-0.9} = \frac{19}{0.1} = \boxed{190}$$

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RELAY 3 Category: GREGORIAN CALENDAR PLAYER 2

Let k be the number you receive from Player 1.

How many of the years from 2018 to 2018 + k (counting both endpoints) are leap years?

Pass your answer to PLAYER 3.

Player 1: 190

$2018 + 190 = 2208$

2020 and 2208 are both divisible by 4

The number of years divisible by 4 is:

$\frac{2208-2020}{4} + 1 = \frac{188}{4} + 1 = 47 + 1 = 48$

However, 2100 and 2200 are NOT leap years,

so the answer is $\boxed{46}$

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RELAY 3 Category: COIN PROBLEM PLAYER 3

Let n be the number that you receive from PLAYER 2.

I have some combination of dimes, nickels, and pennies. I have six coins total with a combined value of exactly n cents.

What (in cents) is total value of the dimes minus the total value of the pennies?

Pass your answer to PLAYER 4.

Player 2: 46

46¢ is 4 dimes, 1 nickel, 1 penny,

and this is the only way to do it, since I'm using as many dimes as possible, and then as many nickels as possible, so this the minimum number of coins.

40 − 1 = 39

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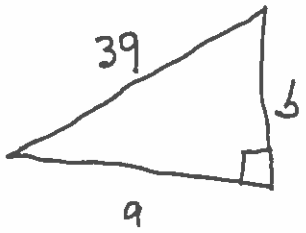
RELAY 3 Category: RIGHT TRIANGLES PLAYER 4

The number you receive from PLAYER 3 is the hypotenuse of a right triangle with all integer side lengths.

What is the perimeter of this triangle?

Run your answer to the front.

Player 3: 39



$a^2 + b^2 = 39^2$

$39 = 3 \cdot 13$

which recalls the "famous" 5-12-13 right triangle

$(3 \cdot 5)^2 + (3 \cdot 12)^2 = (3 \cdot 13)^2$

sides are 15, 36, 39 and $15 + 36 + 39 = \boxed{90}$

This, in fact, is the <u>only</u> way to get a hypotenuse of 39, but you wouldn't need to prove this to get the answer.